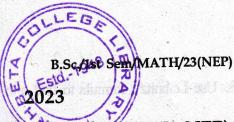
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1st Semester Examination (CCFUP: NEP)

## **MATHEMATICS**

Paper: MI 1-T (Minor)

(Calculus, Geometry and Ordinary Differential Equation)

Time: Three Hours Full Marks: 60

> The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

## Group - A

Answer any ten of the following:  $2 \times 10 = 20$ 

- 1. Show that the point of inflexion of the curve  $y^2 = (x-a)^2(x-b)$  lies on the line 3x+a=b.
- 2. Find the total length of the curve  $x^2(a^2-x^2)=8a^2x^2$ .
- 3. Find the singular solution of  $y = x \frac{dy}{dx} \left(\frac{dy}{dx}\right)^2$ .
- 4. Find the nature of the conic  $11x^2 + 4xy + 14y^2 26x$ -32 y + 23 = 0. to see in painting and the P.T.O.

- 5. Use Leibnitz's formula to find *n*-th derivatives of  $x^3 \log x$ .
- 6. Show that the equation  $(x^3 3x^2y + 2xy^2)dx (x^3 2x^2y + y^3)dy = 0$  is exact.
- 7. Find the equation of the sphere through the circle  $x^2 + y^2 + z^2 = 25$ , x + 2y z + 2 = 0 and the point (1, 1, 1).
- 8. Find the equation of the cylinder whose generators are parallel to x-axis and intersect the curve  $ax^2 + by^2 + cz^2 = 1$ , lx + my + nz = p.
- 9. If  $I_n = \int_0^{\frac{\pi}{4}} \tan^n x dx$ , *n* being a positive integer greater than 1, then prove that  $I_n = \frac{1}{n-1} I_{n-2}$ .
- 10. What does the equation  $4x^2 + 2\sqrt{3}xy + 2y^2 = 1$  become when the axes are rotated through an angle  $\frac{\pi}{6}$ ?
- 11. Show the plane z 1 = 0 which intersects the ellipsoid  $\frac{x^2}{48} + \frac{y^2}{12} + \frac{z^2}{4} = 1$  is an ellipse. Determine the semi-axis.
- 12. Find the integrating factor of  $(1+x^2)\frac{dy}{dx} + y = \tan^{-1} x$ .

- 13. Find the radius of curvature of  $x^3 + y^3 = 3axy$  at the point  $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ .
- 14. Find the point of intersection of the lines  $r\cos(\theta-\alpha)=p$  and  $r\cos(\theta-\beta)=p$ .
- 15. Find the differential equation of all parabolas having their axes parallel to y-axis.

## Group - B

Answer any *four* of the following:  $5 \times 4 = 20$ 

16. Solve 
$$\left(xy^2 - e^{\frac{1}{x^3}}\right) dx - x^2 y dy = 0.$$

- 17. The area enclosed between the arc of the parabola  $y^2 = 4ax$  from the vertex to one extremities of the latus rectum is revolved about the corresponding chord. Find the volume of the spindle thus generated.
- 18. If  $I_n = \int_0^{\pi/2} x^n \sin x \, dx$  and n > 1, show that

$$I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}.$$

Hence find the value of  $\int_0^{\pi/2} x^5 \sin x \, dx$ .

- 19. Determine the values of a, b such that  $\lim_{x\to 0} \frac{x(1+a\cos x)-b\sin x}{x^3} = 1.$
- 20. Reduce the equation to its canonical form and determine the nature of the conic

$$x^2 + 4xy + 4y^2 - 20x + 10y - 50 = 0.$$

21. Find the equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0), (0, 0, 1) and which touches the plane 2x+2y-z=15.

## Group - C

Answer any *two* of the following:  $10 \times 2 = 20$ 

22. (i) If CP, CD be a pair of conjugate semi diameters of an ellipse, prove that the radius of curvature at P is CD<sup>3</sup>/ab where a and b are the length of the semi axes.

(ii) Solve: 
$$(1+y^2)dx - (\tan^{-1} y - x)dy = 0$$
.

23. (i) Find the ranges of values of x for which the curve  $y = x^4 - 6x^3 + 12x^2 + 5x + 7$  is concave upwards or downwards. Also determine the points of inflexion.

- (ii) Solve the differential equation  $y = x 2ap + ap^2$ . Find also its singular solution and interpret it geometrically.
- 24. (i) Evaluate  $\lim_{x\to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x}}$ .
  - (ii) Establish the reduction formula for  $\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x \, dx; \quad m, \quad n \text{ being positive integer}$  greater than 1. Hence calculate  $\int_0^{\frac{\pi}{2}} \sin^5 x \cos^6 x \, dx.$
- 25. (i) Find the equation of the cylinder whose generators are parallel to the straight line  $\frac{x}{-1} = \frac{y}{2} = \frac{z}{3}$  and whose guiding curve is  $x^2 + y^2 = 9$ , z = 1.
  - (ii) Show that the condition that the straight line  $\frac{l}{r} = a\cos\theta + b\sin\theta \quad \text{may touch the circle}$  $r = 2k\cos\theta \text{ is } b^2k^2 + 2ak = 1.$