B.Sc./1st Sem/MATH/23(NEP)

2023

1st Semester Examination (CCFUP : NEP)

MATHEMATICS

Paper : MI 1-T (Minor)

(Calculus, Geometry and Ordinary Differential Equation)

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any *ten* of the following : 2×10=20

1. Show that the point of inflexion of the curve $y^2=(x-a)^2(x-b)$ lies on the line $3x+a=b$.

2. Find the total length of the curve $x^2\left(a^2-x^2\right)=8a^2x^2$.

3. Find the singular solution of $y=x\frac{dy}{dx}-\left(\frac{dy}{dx}\right)^2$.

4. Find the nature of the conic $11x^2+4xy+14y^2-26x-32y+23=0$.

P.T.O.

5. Use Leibnitz's formula to find n-th derivatives of $x^3 \log x$.

6. Show that the equation $\left(x^3-3x^2y+2xy^2\right)dx-\left(x^3-2x^2y+y^3\right)dy=0$ is exact.

7. Find the equation of the sphere through the circle $x^2+y^2+z^2=25$, $x+2y-z+2=0$ and the point (1, 1, 1).

8. Find the equation of the cylinder whose generators are parallel to x-axis and intersect the curve $ax^2+by^2+cz^2=1$, $lx+my+nz=p$.

9. If $I_n=\int_0^{\frac{\pi}{4}}\tan^n x\,dx$, n being a positive integer greater than 1, then prove that $I_n=\frac{1}{n-1}-I_{n-2}$.

10. What does the equation $4x^2+2\sqrt{3}xy+2y^2=1$ become when the axes are rotated through an angle $\frac{\pi}{6}$?

11. Show the plane $z-1=0$ which intersects the ellipsoid $\frac{x^2}{48}+\frac{y^2}{12}+\frac{z^2}{4}=1$ is an ellipse. Determine the semi-axis.

12. Find the integrating factor of $\left(1+x^2\right)\frac{dy}{dx}+y=\tan^{-1}x$.

13. Find the radius of curvature of $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$.

14. Find the point of intersection of the lines $r\cos(\theta - \alpha) = p$ and $r\cos(\theta - \beta) = p$.

15. Find the differential equation of all parabolas having their axes parallel to y-axis.

Group - B

Answer any ***four*** of the following : 5×4=20

16. Solve $\left(xy^2 - e^{\frac{1}{x^3}}\right)dx - x^2 y\,dy = 0$.

17. The area enclosed between the arc of the parabola $y^2 = 4ax$ from the vertex to one extremities of the latus rectum is revolved about the corresponding chord. Find the volume of the spindle thus generated.

18. If $I_n = \int_0^{\pi/2} x^n \sin x\,dx$ and $n > 1$, show that

$$I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}.$$

Hence find the value of $\int_0^{\pi/2} x^5 \sin x\,dx$.

P.T.O.

19. Determine the values of a, b such that

$$\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1.$$

20. Reduce the equation to its canonical form and determine the nature of the conic

$$x^2+4xy+4y^2-20x+10y-50=0.$$

21. Find the equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0), (0, 0, 1) and which touches the plane $2x+2y-z=15$.

Group - C

Answer any ***two*** of the following : 10×2=20

22. (i) If CP, CD be a pair of conjugate semi diameters of an ellipse, prove that the radius of curvature at P is $\frac{CD^3}{ab}$ where a and b are the length of the semi axes. 6

(ii) Solve : $\left(1+y^2\right)dx-\left(\tan^{-1}y-x\right)dy=0$. 4

23. (i) Find the ranges of values of x for which the curve $y=x^4-6x^3+12x^2+5x+7$ is concave upwards or downwards. Also determine the points of inflexion. 5

(ii) Solve the differential equation $y = x - 2ap + ap^2$. Find also its singular solution and interpret it geometrically. 5

24. (i) Evaluate $\lim_{x \to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x}}$. 5

(ii) Establish the reduction formula for $\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x \, dx$; m, n being positive integer greater than 1. Hence calculate $\int_0^{\frac{\pi}{2}} \sin^5 x \cos^6 x \, dx$.

5

25. (i) Find the equation of the cylinder whose generators are parallel to the straight line $\frac{x}{-1} = \frac{y}{2} = \frac{z}{3}$ and whose guiding curve is $x^2 + y^2 = 9$, $z = 1$. 5

(ii) Show that the condition that the straight line $\frac{l}{r} = a\cos\theta + b\sin\theta$ may touch the circle $r = 2k\cos\theta$ is $b^2k^2 + 2ak = 1$. 5
