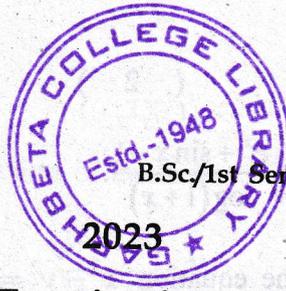


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B.Sc./1st Sem/MATH/23(NEP)

1st Semester Examination (CCFUP : NEP)

MATHEMATICS

Paper : MJ A1-T (Multidisciplinary Major)

(Calculus, Geometry and
Ordinary Differential Equation)

Full Marks : 60

Time : Three Hours

The figures in the margin indicate full marks.

*Candidates are required to give their answers
in their own words as far as practicable.*

Group - A

Answer any *ten* questions : $2 \times 10 = 20$

1. If $y = \tan^{-1} x$, show that

$$(1 + x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0.$$

2. Show that the curve $y = x^3$ has a point of inflexion at the origin.

3. Find the envelope of the family of straight lines

$$y = mx + \sqrt{a^2m^2 + b^2}, \quad m \text{ being the parameter.}$$

P.T.O.

4. Evaluate : $\lim_{x \rightarrow 0} \frac{e^x + \sin x - 1}{\log(1+x)}$.

5. Transform the equation $x^2 - y^2 = a^2$ if the axes are rotated through an angle 45° .

6. Find the values of a and g for which the curve $ax^2 + 8xy + 4y^2 + 2gx + 4y + 1 = 0$ represents a conic having infinitely many centres.

7. Find the points on the conic $\frac{l}{r} = 1 - \cos\theta$, which has smallest radius vector.

8. Find the centre and radius of the circle $x^2 + y^2 + z^2 = 49$, $2x - y + 3z = 14$.

9. What does the equation $x^2 + y^2 = 4$ represent in 3-dimensional geometry?

10. Find the order and degree of the differential equation

$$\frac{dy}{dx} + y^{\frac{3}{2}} = \cos x.$$

11. Solve : $\frac{dy}{dx} = 1 + e^{x+y}$.

12. If $J_n = \int_0^{\frac{\pi}{4}} \tan^n x dx$, prove that

$$J_n = \frac{1}{n-1} - J_{n-2}.$$

13. Find the whole area of the curve

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

14. Find the volume of the solid generated by revolving $x^2 + y^2 = a^2$ about the x -axis.

15. What do you mean by integrating factor? Can it be applied to solve the equation

$$xdy + ydx = k(xdy - ydx)?$$

Group - B

Answer any *four* questions : $5 \times 4 = 20$

16. Find the ranges of values of x for which $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upwards or downwards. Find also its points of inflexion, if any.

17. Find the envelope of the circles described on the radii vector of the circles $x^2 + y^2 = 2ax$ as diameter.

18. Find the area bounded by the curves $y^2 = 4(x+1)$ and $y^2 = -4(x-1)$.

19. Find the volume of the solid obtained by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line.

P.T.O.

(4)

20. Find the equation of the cone with vertex $(0, 0, 0)$ and the guiding curve is $x^2 + y^2 = 4, z = 2$.
21. Solve the equation :

$$y^2 dx + \left(x - \frac{1}{y} \right) dy = 0.$$

Group - C

Answer any *two* questions : $10 \times 2 = 20$

22. (i) Find the equation of the generators of the hyperboloid $x^2 + 4y^2 - 9z^2 = 25$ at the point $(3, 2, 0)$. 5

(ii) Solve : $\left(1 + e^{\frac{x}{y}} \right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y} \right) dy = 0.$ 5

23. (i) Find the condition that the line $\frac{l}{r} = a \cos \theta + b \sin \theta$ may touch the conic $\frac{l}{r} = 1 + e \cos \theta.$ 5
- (ii) Reduce the following equation to its canonical form :

$$4x^2 + 4xy + y^2 - 12x - 6y + 5 = 0. \quad 5$$

24. (i) Find the length of one arch of the cycloid
 $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$. 5

- (ii) Find the constants a , b so that

$$\lim_{x \rightarrow 0} \frac{x(1 + a \cos x) - b \sin x}{x^3} = 1. \quad 5$$

25. (i) Solve : $\frac{dy}{dx} + \frac{1}{x} \cdot \sin 2y = x^3 \cos^2 y$. 5

- (ii) Find the surface area of the solid formed by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about its minor axis. 5