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B.Sc./1st Sem/MATH/23(NEP)

2023

1st Semester Examination (CCFUP : NEP)

MATHEMATICS

Paper : MJ A1-T (Multidisciplinary Major)

(Calculus, Geometry and Ordinary Differential Equation)

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any *ten* questions : 2×10=20

1. If $y = \tan^{-1} x$, show that

$$\left(1+x^2\right)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0.$$

2. Show that the curve $y = x^3$ has a point of inflexion at the origin.

3. Find the envelope of the family of straight lines $y = mx + \sqrt{a^2m^2 + b^2}$, m being the parameter.

P.T.O.

4. Evaluate : $\lim_{x\to 0}\dfrac{e^x+\sin x-1}{\log(1+x)}$.

5. Transform the equation $x^2-y^2=a^2$ if the axes are rotated through an angle 45°.

6. Find the values of a and g for which the curve $ax^2+8xy+4y^2+2gx+4y+1=0$ represents a conic having infinitely many centres.

7. Find the points on the conic $\dfrac{l}{r}=1-\cos\theta$, which has smallest radius vector.

8. Find the centre and radius of the circle $x^2+y^2+z^2=49,\ 2x-y+3z=14$.

9. What does the equation $x^2+y^2=4$ represent in 3-dimensional geometry?

10. Find the order and degree of the differential equation

$$\frac{dy}{dx}+y^{\frac{3}{2}}=\cos x.$$

11. Solve : $\dfrac{dy}{dx}=1+e^{x+y}$.

12. If $J_n=\int_0^{\frac{\pi}{4}}\tan^n x\,dx$, prove that

$$J_n=\frac{1}{n-1}-J_{n-2}.$$

13. Find the whole area of the curve

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$

14. Find the volume of the solid generated by revolving $x^2+y^2=a^2$ about the x-axis.

15. What do you mean by integrating factor? Can it be applied to solve the equation

$$xdy+ydx=k\left(xdy-ydx\right)?$$

Group - B

Answer any ***four*** questions : 5×4=20

16. Find the ranges of values of x for which $y=x^4-6x^3+12x^2+5x+7$ is concave upwards or downwards. Find also its points of inflexion, if any.

17. Find the envelope of the circles described on the radii vector of the circles $x^2+y^2=2ax$ as diameter.

18. Find the area bounded by the curves $y^2=4(x+1)$ and $y^2=-4(x-1)$.

19. Find the volume of the solid obtained by revolving the cardioid $r=a(1+\cos\theta)$ about the initial line.

P.T.O.

20. Find the equation of the cone with vertex (0, 0, 0) and the guiding curve is $x^2+y^2=4,\ z=2$.

21. Solve the equation :

$$y^2dx+\left(x-\frac{1}{y}\right)dy=0.$$

Group - C

Answer any *two* questions : 10×2=20

22. (i) Find the equation of the generators of the hyperboloid $x^2+4y^2-9z^2=25$ at the point (3, 2, 0). 5

(ii) Solve : $\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0.$ 5

23. (i) Find the condition that the line $\frac{l}{r}=a\cos\theta+b\sin\theta$ may touch the conic $\frac{l}{r}=1+e\cos\theta.$ 5

(ii) Reduce the following equation to its canonical form :

$$4x^2+4xy+y^2-12x-6y+5=0.$$ 5

24. (i) Find the length of one arch of the cycloid $x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta)$. 5

(ii) Find the constants a, b so that

$$\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1.$$ 5

25. (i) Solve : $\frac{dy}{dx} + \frac{1}{x}.\sin 2y = x^3 \cos^2 y.$ 5

(ii) Find the surface area of the solid formed by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about its minor axis. 5