B.Sc./2nd Sem (H)/MTM/24(CBCS)

2024

2nd Semester Examination

MATHEMATICS (Honours)

Paper : C 4-T

[Differential Equations and Vector Calculus]

[CBCS]

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any *ten* questions : 2×10=20

   (a) Define Wronskian.

   (b) Show that x^2 and $x\,|x|$ are linearly independent on $-\infty < x < \infty$.

   (c) Find the values of a and b for which all real solutions of the equation $y'' + 2ay' + by = \cos x$ (where a and b are real constants) will be periodic.

   (d) If $y_p(x) = x\cos 2x$ be the particular solution of the differential equation $y'' + \alpha y = -4\sin 2x$ then find the value of α.

P.T.O.

(e) Obtain a linear differential equation with real coefficients that is satisfied by the function

$$y = 3e^{-x} \sin 3x.$$

(f) Reduce the differential equation

$$(x+a)^2 \frac{d^2y}{dx^2} - 4(x+a)\frac{dy}{dx} + 6y = x$$

to a differential equation with constant coefficients.

(g) Show that the vectors $4\hat{i} + 2\hat{j} + \hat{k}$, $2\hat{i} - \hat{j} + 3\hat{k}$ and $8\hat{i} + 7\hat{k}$ are co-planar.

(h) If $\vec{r}.d\vec{r} = 0$ then find $|\vec{r}|$.

(i) If $\vec{r}(t) = 2\hat{i} - \hat{j} + 2\hat{k}$ when $t = 2$

$= 4\hat{i} - 2\hat{j} + 3\hat{k}$ when $t = 3$

Find the value of $\int_2^3 \vec{r}.\frac{d\vec{r}}{dt}\,dt.$

(j) Absolute value of vector triple product is the volume of a parallelepiped. Justify.

(k) Differentiate the singular point and regular singular point.

(l) Discuss the singularity of the differential equation

$$x^2(x-2)^2 \frac{d^2y}{dx^2} + 2(x-2)\frac{dy}{dx} + (x+1)y = 0$$

(m) Explain saddle point of a plane autonomous system.

(n) Why linear combination of two independent solutions are also a solution of a differential equation?

(o) Factorise $\left[xD^2+(x-1)D-1\right]y=x^2$ and reduce it to a first order differential equation.

2. Answer any ***four*** questions : 5×4=20

(a) Solve $x^2\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}-3y=x^2\log x$.

(b) Solve by the method of variation of parameters

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^2e^x.$$

(c) Show that $\left[\vec{\beta}\times\vec{\gamma},\, \vec{\gamma}\times\vec{\alpha},\, \vec{\alpha}\times\vec{\beta}\right]=\left[\vec{\alpha},\, \vec{\beta},\, \vec{\gamma}\right]^2$.

(d) Evaluate $\dfrac{d^2}{dt^2}\left\{\left(\vec{r}\times\dfrac{d\vec{r}}{dt}\right)\times\dfrac{d^2\vec{r}}{dt^2}\right\}$ where $\vec{r}$ is a vector function of t.

(e) If $\vec{F}=xy\hat{i}-z\hat{j}+x^2\hat{k}$. Evaluate $\int_C \vec{F}\times d\vec{r}$, where $C: x=t,\ y=2t,\ z=t^3;\ t:0\to 1$.

(f) Given that $y=x$ is a solution of $\left(x^2-1\right)\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}+2y=0$, find the linearly independent solutions by reducing the order. Write the general solution.

P.T.O.

3. Answer any *two* questions : 10×2=20

(a) Consider the autonomous equation

$$\frac{dx}{dt} = 10x + x^2$$
$$\frac{dy}{dt} = 20y + x^3$$

(i) Determine the type of the critical point (0, 0).

(ii) Obtain the differential equation of the paths and find its general solution.

(iii) Obtain the differential equation of the paths of the corresponding reduced linear system and find its general solution. Determine the stability of the critical point. Make a sketch of the paths in the phase plane.

(b) (i) If $\vec{r}(t) = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$, prove that

$$\int_1^2 \vec{r} \times \frac{d^2\vec{r}}{dt^2} dt = -14\hat{i} + 75\hat{j} - 15\hat{k}.$$ 5

(ii) Solve by the method of undetermined coefficients.

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = x^2 + \sin x.$$ 5

(c) Use the method of Frobenius to find solution near $x=0$ of the differential equation

$$x\frac{d^2y}{dx^2}-\left(x^2+2\right)\frac{dy}{dx}+xy=0.$$ 10

(d) (i) Find the power series solution in power of the following initial value problem

$$\left(x^2-1\right)\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+xy=0,\ y(0)=4,$$

$y'(0)=6.$ 6

(ii) The acceleration of a particle at any time $t\geq 0$ is given by

$$\vec{a}=\frac{d\vec{r}}{dt}=(25\cos 2t)\hat{i}+(16\sin 2t)\hat{j}+(9t)\hat{k}.$$

Find $\vec{r}$. 4