B.Sc./6th Sem (G)/MATH/24(CBCS)

2024

6th Semester Examination

MATHEMATICS (General)

Paper : DSE 1B/2B/3B-T

[CBCS]

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

[Mechanics]

Group - A

Answer any ***ten*** questions : 2×10=20

1. If a particle describes a circle of radius a about the pole then what will be its radial and transversal velocities?

2. The greatest height attained by a projectile is $\frac{1}{4}$th of its range. Find the angle of projection.

3. What do you understand by impulse of a force and an impulsive force?

4. Find the resultant of forces equal to the weights 5 kg and 3 kg respectively acting at an angle 60°.

P.T.O.

5. Define stable and unstable equilibrium.

6. If the tangentail and normal accelerations of a particle moving along a plane curve are equal, then find an expression for its velocity.

7. Define statistical friction and dynamical friction.

8. Masses 2, 3, 4, 5 and 7 lb are placed at the four angular points and the centre of the square. Find the centre of gravity of the system.

9. By what factor should the length of a simple pendulum be changed if the period of vibration tripled?

10. A stone of mass 12 kg is dropped from the height of 60 meters. Find the potential energy and kinetic energy of the stone when it travelled a distance of 24 meters.

11. Prove that the change in kinetic energy is equal to the work done by any acting force.

12. What do you mean by angle of friction and co-efficient of friction?

13. State theorem of Pappus or Guldin. Use the theorem find the C.G. of a semi-circular arc.

14. A simple pendulum of 50 cm long is making 100 oscillation in 2 minutes. Find the acceleration due to gravity.

15. The velocities of a particle along and perpendicular to radius vector from a fixed origin are λr^2 and $\mu\theta^2$. What are laws of motion?

Group - B

Answer any ***four*** questions : 5×4=20

16. An elastic string without weight of which the unstretched length is l and the modulus of elasticity is the weight of n is suspended by one end and a mass of m is attached to other. Show that the time of vertical oscillation is $2\pi\sqrt{\frac{ml}{ng}}$.

17. A body of mass M is propelled in a straight line by an engine producing energy at a constant rate P against a resistance kv^2 where v is the velocity and k is a constant. Show that the space described from rest is given by $\frac{3sk}{M} = -\log\left(1-\frac{kv^3}{P}\right)$.

18. A particle is projected from the earth's surface vertically upwards with a velocity V. If h and H are the greatest heights attained by the particle moving under uniform and variable accelerations respectively, show that $\frac{1}{h}-\frac{1}{H}=\frac{1}{R}$, where R is the radius of the earth.

19. The moments of a system of forces about the points $(0, 0)$, $(a, 0)$ and $(0, a)$ are aw, $2aw$, $3aw$ respectively. Find the components of their resultant parallel to co-ordinate axes and the equation of its line of action.

20. A gun of mass M fires a shell of mass m horizontally

P.T.O.

and the energy of explosion is such as would be sufficient to project the shell vertically to a height *h*. Show that the velocity of the recoil of the gun is $\sqrt{\frac{2m^2gh}{M(M+m)}}$.

21. A hemispherical shell rests on a rough inclined plane whose angle of friction is λ. Show that the inclination of the plane base of the rim to the horizon can not be greater than $\sin^{-1}(2\sin\lambda)$.

Group - C

Answer any ***two*** questions : 10×2=20

22. (i) An elastic string whose natural length is equal to that of a rod is attached to the rod at both ends and suspended by its middle point. The rod is of negligible weight and carries a weight W at its middle point. The system starts from rest when the string and the rod are horizontal. Show that the rod will sink until the strings are inclined to the horizon at an angle θ given by the equation $\cot^3\left(\frac{\theta}{2}\right)-\cot\left(\frac{\theta}{2}\right)=2W$, where the modulus of elasticity of the string is hW.

(ii) Prove that the work-done in raising a body up a smooth inclined plane is the same as the work-done in lifting the body through the vertical height of the plane. 6+4

23. (i) A thin straight smooth tube is made to revolve upwards with a constant angular velocity ω in a vertical plane about one extremity O, when it is in a horizontal position, a particle is at rest in it at a distance 'a' from the fixed end O. Find the distance of the particle from O after any time t.

Show further that if ω is very small, the particle will reach O in time $\left(\dfrac{6a}{g\omega}\right)^{1/3}$.

(ii) A curve is described by a particle having a constant acceleration in a direction inclined at a constant angle to the tangent. Show that the curve is an equiangular spiral. 7+3

24. (i) Explain force of friction and angle of friction.

(ii) A uniform ladder of weight W' rests on a rough horizontal ground against a smooth vertical wall inclined at an angle α to the horizon. Prove that a man of weight W can climb to the top of the ladder without ladder slipping if $\dfrac{W'}{W} > \dfrac{2(1-\mu\tan\alpha)}{2\mu\tan\alpha - 1}$, μ being coefficient of friction.

3+7

25. Prove that the path of a projectile in vacuum is a parabola. Also find the length of its latus rectum. 10

P.T.O.

OR

[Linear Programming]

Group - A

Answer any ***ten*** from the following : 2×10=20

1. Find the extreme point of the set

$$S=\{(x, y):\ x^2+y^2 \leq 16\}.$$

2. Define degenerate solution.

3. Write the standard form of L.P.P.

4. Write the fundamental theorem of L.P.P.

5. Give an example of an L.P.P. which has no feasible regions of solutions.

6. Find a basis for E^3 which contains the vectors (1,2,3) and (2,1,0).

7. Find the value of k for which the vectors (k,2,8), (1,0,4) and (1,k,4) are linearly dependent.

8. Determine the set X is convex or not where

$$X=\{(x_1, x_2): x_1x_2 \leq 1, x_1, x_2 \geq 0\}.$$

9. Define slack and surplus variable.

10. Discuss whether the set A consisting on the straight line $y=mx+c$ is convex and corresponding convex hull.

11. A man rides his motorcycle at the speed of 50 km/hour. He has to spend Rs. 2 per km on petrol. If he rides it at a faster speed of 80 km/hour, the petrol cost increases to Rs. 3 per km. He has atmost Rs. 120 to spend on petrol and one hour's time. He wishes to find the maximum distance that he can travel. Express this problem as a linear programming problem.

12. If the objective function of an LPP assumes its optimal value at more than one extreme point, then prove that every convex combination of these extreme points give the optimal solution.

13. What do you mean by cycling in an LPP? How it can be resolved?

14. Show that the intersection of two convex sets is also convex.

15. Find a basic feasible solution of the system :

$$x_1 + 2x_3 = 1;\ x_2 + x_3 = 4;\ x_1, x_2, x_3 \geq 0.$$

Group - B

Answer any *four* from the following : 5×4=20

16. Prove that the dual of the dual is the primal.

17. Examine whether the following set S is convex :

$$S = \{(x, y) : 2x + y \geq 0,\ x + 2y \leq 80,\ x + y \leq 50,\ x, y \geq 0\}.$$

P.T.O.

18. How many basic solutions are there in the following set of equations?

$$2x_1 - 5x_2 + x_3 + 3x_4 = 4,$$

$$3x_1 - 10x_2 + 2x_3 + 6x_4 = 12,$$

Find all basic solutions.

19. Show that the set of all feasible solutions of a linear programming problem is a convex set.

20. Prove that if a constraint of a primal problem is an equation then the corresponding dual variable is unrestricted in sign.

21. Prove that dual of dual is primal.

Group - C

Answer any *two* from the following : 10×2=20

22. (i) Food X contains 6 units of vitamin A per gram and 7 units of vitamin B per gram and cost 12 paise per gram. Food Y contains 8 units of vitamin A per gram and 12 units of vitamin B per gram and cost 20 paise per gram. The daily minimum requirement of vitamin A and B are 100 units and 120 units respectively. In finding the minimum cost of product mix, formulate the problem as a linear programming problem.

(ii) Solve graphically the following L.P.P. problem :

Minimize $Z = -2x + 7y$

Subject to $3x + 2y \le 17$

$-2x + 3y \le 6$

$y \ge 1$

$x, y \ge 0.$ 5+5

23. (i) Solve the following L.P.P. using simplex method :

Maximize $Z = x_1 + x_2 + 3x_3$

Subject to $3x_1 + 2x_2 + x_3 \le 3,$

$2x_1 + x_2 + 2x_3 \le 2,$

$x_1, x_2, x_3 \ge 0.$

(ii) Examine whether the set is convex or not

$$X = \{(x_1, x_2), x_1 \ge 2, x_2 \le 3, x_1, x_2 \ge 0\}.$$ 7+3

24. Solve the following LPP using two phase method :

Maximize $Z = 2x_1 + x_2 - x_3$

Subject to $4x_1 + 6x_2 + 3x_3 \le 8,$

$2x_1 - 6x_2 + 3x_3 \le 1,$

$2x_1 + 3x_2 - 5x_3 \ge 4,$

$x_1, x_2, x_3 \ge 0.$

P.T.O.

25. Use Charne's Big M-method to solve the following LPP :

Minimize $Z = 6x_1 + 4x_2 + 3x_3$

Subject to $4x_1 + 5x_2 + 3x_3 \geq 40;$

$2x_1 + x_2 + 6x_3 \geq 50;$

$3x_1 + 4x_2 + 2x_3 \geq 50;$

$x_1, x_2, x_3 \geq 0.$

OR

[Numerical Methods]

Group - A

Answer any *ten* questions : 2×10=20

1. Find the function $f(x)$ whose first difference is e^x.

2. Find the value of $(1+\Delta)(1-\nabla)$ on $f(x)$ where the symbols have usual meaning.

3. Write down the Newton-Raphson iteration formula relating the nth and $(n+1)$th approximation to a real root of $x^2 - 5x + 2 = 0$.

4. How does the trapezoidal rule approximate the area under the curve defined in an interval $[a, b]$?

5. State central difference formulae mentioning the assumptions involved.

6. Show that second differences are zero for the linear function $f(x) = 2x + 5$.

7. Find the value of y at $x = 2$ by the Lagrange's interpolation formula from the table.

x	1	3
y	3	12

P.T.O.

8. Find the second approximation to the real root of $f(x) = x^3 - 4x - 1 = 0$ lying between 2 and 3 by the Regula-Falsi method.

9. State the Newton's forward interpolation formula mentioning the assumptions involved.

10. Establish the relation between shift operator and difference operator.

11. Examine whether Gauss-Seidal iteration method is applicable to solve the following system of equations : $5x + 2y - z = 1$, $x + 5y + 2z = 3$ and $-2x + y + 6z = 5$.

12. Construct a difference table upto second order of $y = x^2$ for $x = 1,3,4,5,7,9$.

13. If $f(x) = ax$, show that $\left(E^{-1} + E\right) f(x) = 2f(x)$.

14. Write the formula for fourth order Runge-Kutta method to find the solution of IVP : $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$.

15. Given $f(-1) = 1$, $f(0) = 1$, $f(2) = -5$, find $f(1)$ using the Lagrange's interpolation formula.

Group - B

Answer any ***four*** questions : 5×4=20

16. Find a real root of the equation $\sin x = 5x - 2$, using Regula-Falsi method correct to three decimal places.

17. Derive Simpson's 1/3rd rule for numerical integration $\int_a^b f(x)dx$. State its geometrical significance.

18. Describe "Fixed point iteration method". Mention the condition for the convergence. When does the method fails?

19. Write down the quadratic polynomial by applying the Lagrangian interpolation method, which takes some values as $f(x)$ at $x = -1, 0, 1$ and obtain the integration rule $\int_{-1}^{1} f(x)dx = \frac{1}{3}[f(-1)+4f(0)+f(1)]$.

20. Given $x_1 = x_0 + h$, $x_2 = x_0 + 2h$, $x_3 = x_0 + 3h$, $h > 0$ and $u_3(x) = (x - x_0)(x - x_1)(x - x_2)(x - x_3)$, prove that $\Delta^3 u_3(x) = 4!h^3(x - x_0)$.

21. Compute the value of $f(3.8)$ by the Newton's Backward interpolation formula from the following data :

X	0	1	2	3	4
$F(x)$	1	1.5	2.2	3.1	4.2

Group - C

Answer any ***two*** questions : 10×2=20

22. (a) Find the third iterated solution of the following system of equation by Gauss-Jacobis method : $x + 2y + z = 0$, $2x + 2y + 3z = 3$, $-x - 3y = 2$.

P.T.O.

(b) Given $\frac{dy}{dx} = x^2 + y$ with $y(0) = 1$. By Euler's method determine $y(0.02)$ with step length $h = 0.01$. 5+5

23. (a) Describe the Newton-Raphson's method to determine approximately one simple real root of $f(x) = 0$. Mention the condition of convergence. Apply it to find the real root of the equation $x^2 + 4\sin x = 0$ correct to three decimal places.

(b) Evaluate $\int_0^{\pi/2} \sqrt{\cos x}\, dx$ by Weddle's Rule, correct upto three significant figures taking 6-intervals.

6+4

24. (a) Using predictor corrector formula find $y(2)$ if $y(x)$ is the solution of $y' = \frac{1}{2}(x + y)$ where $y(0) = 2$, $y(0.5) = 2.636$, $y(1) = 3.595$, $y(1.5) = 4.968$.

(b) Show that $\Delta^4 y_0 = y_4 - 4y_3 + 6y_2 - 4y_1 + y_0$. 7+3

25. (a) Compute $y(1.3)$ from $y' = x^2 + y^2$ with $y(1) = 0$, using fourth order Runge-Kutta method with step size $h = 0.3$.

(b) Find the first and second derivatives of the function tabulated below at the point $x = 1.1$.

X	1.0	1.2	1.4	1.6	1.8	2.0
$F(x)$	0.000	0.128	0.544	1.296	2.432	4.000

5+5

OR

[Integer Programming and Theory of Games]

Group - A

Answer any *ten* questions : $2\times10=20$

1. What is mixed integer programming problem?

2. At every stage of branch and bound algorithm, we add constraints. Does this increase the size of the LP solved?

3. Define the mixed strategies in the case of rectangular game.

4. Prove that the set of all optimal strategies of a player form a convex set.

5. What are the differences between integer programming problem and linear programming problem?

6. Define two-person zero sum game.

7. For the following LPP

   Maximize $Z = 7x_1 + 6x_2 + 4x_3$

   Subject to $x_1 + x_2 + x_3 \leq 5$

   $2x_1 + x_2 + 3x_3 \leq 10$

   $x_1, x_2, x_3 \geq 0$

   find the number of basic solutions.

P.T.O.

8. Define analytical definition of saddle point.

9. Solve the game whose payoff matrix is :

Player B

		I	II	III
Player A	I	2	4	5
	II	10	7	9
	III	4	5	6

10. Show that the 2×2 game $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is non-strictly determined, if $a < b$, $a < c$, $d < b$ and $d < c$.

11. What is the principle behind the Gromory Cut?

12. What is the principle behind the branch and bound algorithm of integer programming?

13. Explain the following terms :

    (i) Value of the game

    (ii) Pay-off-matrix

14. What do you understand by Game Theory? Distinguish between pure and mixed strategies for a game.

15. What do you mean by dominance in game theory?

Group - B

Answer any ***four*** questions : 5×4=20

16. Convert the following Game into an LPP :

$$\begin{bmatrix} 8 & 5 & 2 \\ 6 & 1 & -4 \\ 2 & 3 & 3 \end{bmatrix}.$$

17. Solve the following game using graphical method.

$$\begin{bmatrix} 2 & 4 & -2 & 6 & 3 \\ 3 & 7 & 3 & 7 & 4 \end{bmatrix}$$

18. Solve :

Maximize $Z = 3x_1 + 4x_2$

Subject to $3x_1 + 2x_2 \leq 8$

$x_1 + 4x_2 \leq 10$

$x_1, x_2 \geq 0$ and all are integers.

19. Use dominance to reduce the following game problem to 2×2 game and hence find the optimal strategies and the value of the game.

	Player B		
Player A	3	–2	4
	–1	4	2
	2	2	6

P.T.O.

20. Find the range of x and y which will render the entry (2, 2) a saddle point for the game.

Player B

	2	4	5
Player A	10	7	x
	4	y	6

21. Find the solution of the following game :

Player B

	–2	0	0	5
Player A	4	2	1	3
	–4	–3	0	–2
	5	3	–4	2

Group - C

Answer any ***two*** questions : 10×2=20

22. Use Gromory's cutting plane method to find the optimal solution of the L.P.P.

Maximize $Z = 7x_1 + 9x_2$

Subject to $-x_1 + 3x_2 \leq 6$

$7x_1 + x_2 \leq 35$

$x_1, x_2 \geq 0$ and all are integers.

23. Use dominance property to reduce the pay-off matrix given by

Player B

Player A	3	–1	1	2
	–2	3	2	6
	2	–2	–1	1

into a 2×2 matrix and find the mixed strategies for *A* and *B*. Also find the value of the game.

24. Find the optimal solution to the L.P.P.

Maximize $Z = x_1 + x_2$

Subject to $3x_1 + 2x_2 \leq 5;$

$x_2 \leq 2;$

$x_1, x_2 \geq 0$ and all are integers.

25. If the pay-off matrix for the Player A is given by

Player B

Player A	a_{11}	a_{12}
	a_{21}	a_{22}

Solve this game with its value in terms of a_{11}, a_{12}, a_{21}, a_{22}.

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