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Total Pages : 4 B.Sc./4th Sem (H)/PHYS/24(CBCS)

2024

4th Semester Examination

PHYSICS (Honours)

Paper : C 8-T

[Mathematical Physics - III]

[CBCS]

Full Marks : 40 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answer wherever necessary.
Symbols have their usual meaning.

Group - A

Answer any *five* questions. 2×5=10

1. Determine the types of singularities (if any) possessed by the following functions at $z = 0$ and $z = \infty$: $\sin h\left(\frac{1}{z}\right)$ and $\frac{e^z}{z^3}$.

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2. Find the Fourier transform of : $f(t)=\begin{cases} 0, & t<0 \\ Ae^{-\tau t}, & t>0 \end{cases}$

where, $\tau > 0$.

3. Show that A is orthogonal, where

$$A=\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}.$$

4. Obtain the analytic function whose real part is $u(x,y)=e^{x}\cos y$.

5. If λ_1, λ_2 and λ_3 are the eigenvalues of the matrix

$$\begin{bmatrix} -2 & -9 & 5 \\ -5 & -10 & 7 \\ -9 & -21 & 14 \end{bmatrix},$$ find the value of $\lambda_1+\lambda_2+\lambda_3$

6. Prove that $\delta(bt)=\delta(t)/|b|$.

7. Find the values of

$\int_C \frac{1}{Z-a}\,dz$ when $z=a$ point is inside of C and outside of C respectively.

8. Explain the significance of the Convolution Theorem in signal processing.

Group - B

Answer any ***four*** questions. 5×4=20

9. Using Cayley Hamilton theorem find the inverse matrix

$$\begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix}$$

10. Solve, with the help of matrices, the simultaneous equations $x+y+z=3, x+2y+3z=4, x+4y+9z=6$.

11. Find the radii of convergence of the following Taylor series :

(i) $\sum_{n=2}^{\infty}\frac{z^n}{\ln(n)}$, (ii) $\sum_{n=1}^{\infty}\frac{n!z^n}{n^n}$ 2½+2½

12. Derive the Cauchy-Riemann equations in connection with analyticity as a function of complex variables.

13. Find Fourier Cosine transform of

$f(x)=e^{-ax},(a>0, x\geq 0)$.

14. Find the Laurent series of $f(z)=\frac{1}{z(z-2)^3}$ about the singularities $z=0$. Hence verify that $z=0$ is a pole of order 1 and find the residue of $f(z)$ at the pole.

3+1+1

Group - C

Answer any *one* question : 10×1=10

15. (a) Find out the eigenvalues and Eigen vectors of the given Hermitian matrix

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(b) Using contour integration to evaluate the real integral $\int_0^{2\pi} \frac{1}{5-4\sin\theta} d\theta = \frac{2\pi}{3}$. 5+5

16. (a) Find the inverse Fourier transform of the function :

$$F(\omega) = \frac{12+7j\omega-\omega^2}{(\omega^2-2j\omega-1)(-\omega^2+j\omega-6)}$$

(b) Fourier transform of $e^{-|x|}$ is $\sqrt{\frac{2}{\pi}}\left(\frac{1}{1+s^2}\right)$. Using Parseval's identity show that $\int_0^{\infty} \frac{dx}{(x^2+1)^2} = \frac{\pi}{4}$.

6+4

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