

2022

5th Semester Examination  
MATHEMATICS (Honours)

Paper : C 11-T

[Partial Differential Equations and Applications]

[CBCS]

Full Marks : 60

Time : Three Hours

*The figures in the margin indicate full marks.  
Candidates are required to give their answers  
in their own words as far as practicable.*

Group - A

1. Answer any *ten* questions :  $2 \times 10 = 20$

- (a) What is ballistics? Write different types of ballistics.
- (b) Define quasi-linear and semi-linear partial differential equation.
- (c) Find the general solution of second order PDE

$$4 \frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0.$$

- (d) What is the nature of the second order PDE

$$\frac{\partial^2 z}{\partial y^2} - y \frac{\partial^2 z}{\partial x^2} + x^3 z = 0?$$

P.T.O.

(e) Let  $a, b \in \mathbb{R}$  be such that  $a^2 + b^2 \neq 0$ . Then

prove that the Cauchy problem  $a \frac{\partial z}{\partial x} + b \frac{\partial z}{\partial y} = 1$ ,

$x, y \in \mathbb{R}$  with  $z(x, y) = x$  on  $ax + by = 1$  has a unique solution.

(f) Find the characteristic curve of PDE :

$2y \frac{\partial z}{\partial x} + (2x + y^2) \frac{\partial z}{\partial y} = 0$  which is passing through the point  $(0, 0)$ .

(g) Find the equations of the characteristic curves of

the PDE  $(x^2 + 2y) \frac{\partial^2 z}{\partial x^2} + (y^3 - y + x) \frac{\partial^2 z}{\partial y^2}$

$+ x^2(y-1) \frac{\partial^2 z}{\partial x \partial y} + 3 \frac{\partial z}{\partial x} + z = 0$  which are passing

through the point  $x = 1, y = 1$ .

(h) Let  $z(x, t)$  be the equation of  $\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial t^2}$  with

$z(x, 0) = \cos(5\pi x)$  and  $\frac{\partial z}{\partial t}(x, 0) = 0$ . Then

prove that  $z(1, 1) = 1$ .

(i) Show that the solution of the PDE :

$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 0$  is of the form  $f(y/x)$ .

(j) Prove that the partial differential equation

$$x \frac{\partial^2 z}{\partial x^2} + y \frac{\partial^2 z}{\partial y^2} = 0 \text{ is elliptic type for } x < 0, y > 0.$$

(k) Find the two families of surfaces that generate the

$$\text{characteristics of } (3y - 2z) \frac{\partial z}{\partial x} + (z - 3x) \frac{\partial z}{\partial y} = 2x - y.$$

(l) Find the partial differential equation by eliminating arbitrary constants  $a$  and  $b$  from  $z = (x + a)(y + b)$ .

(m) Define apsidal angle and apsidal distance.

(n) Prove that a planet has only a radial acceleration towards the Sun.

(o) Prove that at an apse on a central orbit, the velocity is proportional to the reciprocal of the radius vector.

### Group - B

2. Answer any *four* questions :

5×4=20

(a) A particle moves with a central acceleration  $\mu \div (\text{distance})^2$ ; it is projected with velocity  $v$  at a distance  $R$ . Show that its path is a rectangular hyperbola if the angle of projection is

$$\sin^{-1} \left[ \frac{\mu}{\left\{ VR \left( V^2 - \frac{2\mu}{R} \right)^{1/2} \right\}} \right].$$

P.T.O.

- (b) A spherical raindrop falls through a cloud while accumulating mass at a rate  $\lambda r^2$  where  $r$  is its radius and  $\lambda > 0$ . Find its velocity at time  $t$  if it starts from rest with radius  $a$ .
- (c) Find the integral surface of the PDE,  $x(z+2a)p + (xz+2yz+2ay)q = z(z+a)$ .
- (d) Using the method of separation of variables, solve :  

$$\frac{\partial z}{\partial x} = q \frac{\partial z}{\partial t} + z \text{ where } z(x, 0) = 6e^{-3x}.$$
- (e) Reduce the wave equation  $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$  to canonical form.
- (f) Solve  $z^2 = pqxy$  by Charpit's method.

### Group - C

3. Answer any *two* questions : 10×2=20

- (a) (i) If a point moves on a Curve with constant tangential acceleration such that the magnitudes of the tangential velocity and normal acceleration are in a constant ratio, find the  $(s, \psi)$  equation of the curve.
- (ii) Solve  $(D^3 - 3DD'^2 - 2D'^3)z = \cos(x+2y)$ .
- (b) (i) A particle is projected with velocity  $V$  from

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the cusp of a smooth inverted cycloid down the arc, show that the time of reaching the

vertex is  $2\sqrt{\frac{a}{g}} \tan^{-1} \left[ \sqrt{\frac{4ag}{V}} \right]$ . 4+6

(ii) Using the method of separation of variables, solve the following wave equation described by

$$\text{PDE: } \frac{\partial^2 z}{\partial t^2} = 4 \frac{\partial^2 z}{\partial x^2}$$

$$\text{BCS: } z(0, t) = 0, z(s, t) = 0$$

$$\text{ICS: } z(x, 0) = 0, \left( \frac{\partial z}{\partial t} \right)_{t=0} = 5 \sin \pi x. \quad 5+5$$

(c) (i) Solve the boundary value problem

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}, \quad u(x, 0) = 20, \quad u(0, t) = 0, \\ u(L, t) = 0.$$

(ii) Find the integral surface of the linear PDE

$$2y(z-3) \frac{\partial z}{\partial x} + (2x-z) \frac{\partial z}{\partial y} = y(2x-3) \text{ which}$$

passes through the circle  $x^2 + y^2 = 2x, z = 0$ .

5+5

P.T.O.

(d) (i) Solve two dimensional Laplace's equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

with BCS  $z(0, y) = 0$ ,  $z(l, y) = 0$

and  $z(x, y) \rightarrow 0$  as  $y \rightarrow \infty$

$$z(x, 0) = f(x)$$

(ii) Let  $u(x, y)$  be the solution of the Cauchy

problem  $\frac{\partial u}{\partial y} - x \frac{\partial u}{\partial x} + u = 1$ , where

$-\infty < x < \infty$ ,  $y \geq 0$  and  $u(x, 0) = \sin x$ , then

find  $u(0, 1)$ .

5+5