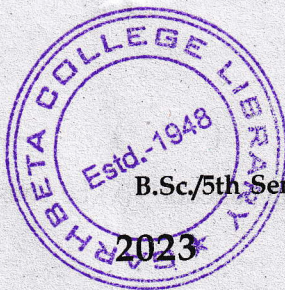


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B.Sc./5th Sem (H)/PHS/23(CBCS)

2023

5th Semester Examination

PHYSICS (Honours)

Paper : C 11-T

[Quantum Mechanics and Applications]

[CBCS]

Full Marks : 40

Time : Two Hours

*The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.*

Group - A

Attempt any **five** questions : $2 \times 5 = 10$

1. At time $t = 0$, a particle is represented by the wave function :

$$\begin{aligned}\varphi(x, 0) &= A(x/a), 0 \leq x \leq a \\ &= A(b-x)/(b-a), a \leq x \leq b \\ &= 0, \text{otherwise}\end{aligned}$$

Find the value of A .

2

P.T.O.

(2)

2. Can the following function $f(x)$ be considered as a wave function?

$$[f(x)]^2 = 4x \exp(-x)$$

Explain.

2.

3. Show that the operator (d/dx) can have only imaginary eigenvalues.

2

4. Suppose wave function of a particle is given by

$$\varphi(x) = A e^{\frac{ip_0 x}{\hbar}}. \text{ Find out the corresponding wave function in momentum space.}$$

2

5. What is zero-point energy? Why can't it be zero?

2

6. Consider three bosons inside an infinite 1-D potential well of dimension "a". Write down the wave function for the ground state and 1st excited state.

1+1

7. What was the conclusion of Stern-Gerlach experiment?

2

8. The state space of a system is described by the orthonormal basis vectors $|e_1\rangle$ and $|e_2\rangle$. Consider the operator : $A = k(|e_1\rangle\langle e_1| - i|e_1\rangle\langle e_2| + i|e_2\rangle\langle e_1|) - |e_2\rangle\langle e_2|$, where k is a real constant. Find the eigenvalues of A .

2

Group - B

Attempt any **four** questions : $5 \times 4 = 20$

9. What do you mean by spin-orbit coupling? Find out the Lande g-factor for 3P_2 . What is the main difference

(3)

between orbital angular momentum and spin angular momentum? 2+2+1

10. Consider a system whose wave function is given by :

$$\Psi(x,0) = \frac{5}{\sqrt{50}}\phi_0(x) + \frac{4}{\sqrt{50}}\phi_1(x) + \frac{3}{\sqrt{50}}\phi_2(x)$$

where $\phi_n(x)$'s are the eigenfunction of Hamiltonian for a harmonic oscillator. Find the average energy of the system and expected number of energy quanta present in the system. 4+1

11. Find the eigenvalues and eigenstates of the component of the spin operator \vec{S} of an electron along the direction of unit vector \hat{n} , assume \hat{n} lies in the XZ -plane. 5

12. Prove that, $\hat{L}_\pm |l, m\rangle = \hbar\sqrt{l(l+1) - m(m\pm 1)} |l, m\pm 1\rangle$

Using this relation, find out expectation value of L_+ for the state 3+2

$$\frac{1}{\sqrt{5}}Y_{20} + \sqrt{\frac{2}{5}}(Y_{2,-1} - Y_{2,1})$$

13. Explain how Sodium D_1 line ($3^2P_{1/2} \rightarrow 3^2S_{1/2}$) and D_2 line ($3^2P_{3/2} \rightarrow 3^2S_{1/2}$) split into four and six components respectively under anomalous Zeeman effect. 5

P.T.O.

14. (a) Consider the wave function $\Psi(x) = A x e^{-\frac{m\omega x^2}{2\hbar}}$,

corresponding to the potential $V(x) = \frac{m\omega^2 x^2}{2}$.

- (i) Find the normalization constant.
- (ii) Find the probability of finding the particle in the classically forbidden region in this state.

[Hint : $\frac{2}{\sqrt{\pi}} \int_0^1 u^2 e^{-u^2} du = 0.214$]

- (b) Find the term symbol of Nitrogen Atom.

1+2+2

Group - C

Attempt any **one** question : $10 \times 1 = 10$

15. Consider a particle of mass m moving in a one-dimensional harmonic oscillator potential.

- (a) Define $\hat{a} = \frac{1}{\sqrt{2m\hbar\omega}} [m\omega\hat{x} + i\hat{p}]$ and

$$\hat{a}^\dagger = \frac{1}{\sqrt{2m\hbar\omega}} [m\omega\hat{x} - i\hat{p}].$$

Find the commutation relation between a and a^\dagger .

2

- (b) Express Hamiltonian of LHO in terms of above two operators.

2

(c) Prove that,

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle, \quad \hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle \quad 3$$

(d) Calculate the uncertainty product for position and momentum operator in the 5th excited state, i.e. $n = 5$. Is it larger or smaller than that for the ground state? Comment. 2+1

16. Consider a hydrogen atom which is in its ground state; the ground state wave function is given by :

$$\Psi(r, \theta, \phi) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}, \text{ where } a_0 \text{ is the Bohr radius.}$$

(a) Find out the most probable distance between the electron and proton. 3

(b) Find out the average distance between the electron and proton. 3

(c) How many degenerate states are there for $n = 3$ state of hydrogen atom? Write down all the degenerate states in $|n, l, m\rangle$ notation. 1+3
