B.Sc./6th Sem (G)/MATH/23(CBCS)

2023

6th Semester Examination

MATHEMATICS (General)

Paper : DSE 1B/2B/3B-T

[CBCS]

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

[Mechanics]

Group - A

Answer any *ten* questions : 2×10=20

1. If a particle moves in a straight line with uniform acceleration, prove that increase (or change) in K.E. is equal to the work done by the acting force.

2. If a bicyclist always works $\frac{1}{10}$ H.P. and goes 12 miles per hour on the level ground, find the resistance of the road.

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3. A particle describes the curve $y = \frac{c}{2}\left(e^{\frac{x}{c}} + e^{-\frac{x}{c}}\right)$ under a force which is always parallel to the positive direction of the y-axis. Find the law of force.

4. Deduce the expression for the radial velocity of a particle moving in a plane curve.

5. If the radial velocity is proportional to the transverse velocity, find the path in polar co-ordinate.

6. If the tangential and normal acceleration of a particle moving in a plane curve are equal, find the expression for the velocity.

7. State Newton's Law of motion.

8. If the velocity of a particle at a distance x from a fixed point O is given by $v^2 = a - bx^2$, where a, b are constant, prove that the motion is simple harmonic and find its period.

9. A particle starts from rest from the top of a smooth inclined plane of a given base. Show that the time of fall is least when the inclination of the plane to the horizon is 45°.

10. Define astatic equilibrium and astatic centre.

11. A system of co-planar forces has the total moment H, $2H$ and $3H$ about the point (0, 0), (0, 1) and (2, 4) respectively. Find the magnitude of the resultant.

12. Define limiting friction and co-efficient of friction.

13. Two forces of magnitudes $3P$, $2P$ respectively have a resultant R. If the first force is doubled, the magnitude of the resultant is doubled. Find the angle between the forces.

14. If two bodies A and B are rigidly joined together and if their weights be W_1 and W_2 and C.G. be G_1 and G_2 respectively, from a fixed point O, find the C.G. of the two combined bodies.

15. Find the C.G. of an arc of a semi-circle.

Group - B

Answer any ***four*** questions : 5×4=20

16. Find the C.G. of the area of the cardiod $r = a(1 + \cos\theta)$.

17. The straight line $4x + 3y = 5$ meets the rectangular axes Ox, Oy at A and B respectively. If the forces X, Y, Z act along the lines OB, OA and AB, find the magnitude of the resultant and the equation of line of action.

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18. A particle executing S.H.M. in a straight line has velocities v_1, v_2, v_3 respectively at distances x_1, x_2, x_3 from the centre of the path. Prove that $x_1^2\left(v_2^2 - v_3^2\right) +$

$x_2^2\left(v_3^2 - v_1^2\right) + x_3^2\left(v_1^2 - v_2^2\right) = 0$.

19. Find the constant force necessary to move a train of mass 150 ton up an incline of 1 of 200 through half a mile in a minute, starting from rest, resistance due to friction being 12 lbs wt. per ton. ($g = 32$ ft/sec^2).

20. A particle moves in a plane such that its acceleration parallel to the x and y are $k^2 a \sin kt$ and $k^2 a \cos kt$ respectively with the initial conditions $x = 0$, $y = -a$, $\frac{dy}{dt} = 0$, $\frac{dx}{dt} = -k.a$ when $t = 0$. Find the equation of the path of the particle.

21. A particle describes the equi-angular spiral $r = ae^{m\theta}$ with a constant velocity. Find the components of the velocity and of the acceleration along the radius vector and perpendicular to it.

Group - C

Answer any ***two*** questions : 10×2=20

22. (i) The couple components of a system of co-planar forces when reduced w.r. to two different bases O and O' are G and G' respectively. Show that the couple component when the system is reduced with respect to the middle point of OO' is $\frac{1}{2}(G + G')$.

(ii) Find the C.G. of the segment of a solid sphere of radius 'a' cut off by a plane situated at a distance 'c' ($< a$) from the centre. 4+6=10

23. (i) Investigate the condition of equilibrium of a particle constrained to rest on a rough surface $f(x, y, z) = 0$ under any given forces acting on the particle.

(ii) The paraboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2z$ is placed with its axis vertical and its vertex uppermost; if μ be the co-efficient of friction, show that a particle will rest on it at any point above its curve of intersection with the cylinder $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \mu^2$. 4+6=10

24. A particle of mass m is projected into the air with velocity u in a direction making an angle α with the horizontal. Find the path of the particle, time of flight and maximum horizontal range. 6+2+2

25. (i) A particle falls under constant gravity, through a distance x, starting from rest. A small resistance per unit mass, equal to K times the square of the speed, acts on the particle. Find the kinetic energy of the particle.

P.T.O.

(ii) A particle is projected horizontally under gravity with velocity $\sqrt{3ga}$ from the lowest point of the inner side of a smooth circular curve of radius a. Find the highest point of the curve reached by the particle. 5+5

OR

[Linear Programming]

Group - A

Answer any ***ten*** from the following : 2×10=20

1. Show that the arbitrary intersection of convex sets is a convex set.

2. Find the extreme points if any of the set $S = \{(x_1, x_2) : |x_1| \leq 1, |x_2| \leq 1\}$.

3. Define Convex hull and convex polyhedron.

4. Find the basic solution with $x_3 = 0$ as non basic variable of the following equations :

   $x_1 + 4x_2 - x_3 = 3;\ 5x_1 + 2x_2 + 3x_3 = 4.$

5. Discuss the advantages and disadvantages of graphical method for solving an LPP.

6. Find the basic feasible solution of the system

   $x_1 + 2x_3 = 1;\ x_2 + x_3 = 4;\ x_1, x_2, x_3 \geq 0.$

7. Express the vector (1,1,1) as a linear combination of three vectors (1,2,3), (4,2,1) and (2,4,2).

8. Show that $X = \{x : x| \leq 2\}$ is a convex set.

P.T.O.

9. Show that the vectors (3,0,2), (7,0,9) and (4,1,2) form a basis for E^3.

10. Define feasible solution and Basic feasible solution.

11. Define slack and surplus variables.

12. In simplex table, how you can determine a redundant constraint equation?

13. If the objective function assumes its optimal value at more than one extreme point, then show that every convex combination of these extreme points also gives the optimal value of the objective function.

14. Give example of simplex in zero and two dimension.

15. Define degenerate and non degenerate basic feasible solution.

Group - B

Answer any ***four*** from the following : 5×4=20

16. Find all the basic solutions of the system

$$2x_1 + x_2 + 4x_3 = 11,$$
$$3x_1 + x_2 + 5x_3 = 14.$$

17. Solve graphically the following LPP

Minimize $Z = x_1 + 3x_2$

Subject to $-x_1 + 2x_2 \le 4;$

$x_1 + x_2 \le 6;$

$x_1 + 3x_2 \ge 9;$

$x_1, x_2 \ge 0.$

18. A company produces two types of goods, A and B that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold while that of type B requires 1 g of silver and 2 g of gold. The company can use at the most 9 g of silver and 8 g of gold. If each unit of type A brings a profit of Rs. 40 and that of type B Rs. 50, find the number of units of each type that the company should produce to maximize profit. Formulate the above LPP and solve it and also find the maximum profit.

19. Find the dual of the following LPP :

Minimize $Z = x_1 + x_2 + x_3$

Subject to $x_1 - 3x_2 + 4x_3 = 5;$

$x_1 - 2x_2 \leq 3;$

$2x_2 - x_3 \geq 0;$

$x_1 x_2 \geq 0;$ x_3 *is unrestricted in sign.*

20. Prove that a basic feasible solution to a linear programming problem corresponds to an extreme of the convext set of feasible solutions.

21. Using Simplex Method solve the following LPP :

Maximize $Z = 60x_1 + 50x_2$

Subject to $x_1 + 2x_2 \leq 40;$

$3x_1 + 2x_2 \leq 60;$

$x_1, x_2 \geq 0$.

P.T.O.

Group - C

Answer any *two* from the following : 10×2=20

22. Find the optimal solution of the following LPP by solving its dual :

$$Maximize\ Z = 2x_1 + 3x_2$$

Subject to $-x_1 + 2x_2 \le 4;$

$x_1 + x_2 \le 6\,;$

$x_1 + 3x_2 \le 9;$

$x_1\, x_2 \ge 0;$

23. (i) A person requires 10, 12 and 12 units of chemical A, B and C respectively. A liquid product contains 3, 2 and 1 units of A, B can C respectively. A dry product contains 1, 2 and 4 units of A, B and C per packet. If the liquid product sells for Rs. 2 per jar and the dry product sells for Re. 1 per packet, then formulate the problem as a linear programming problem.

(ii) Solve graphically the following L.P.P. problem

$$Minimize\ Z = -2x + 7y$$

Subject to $3x + 2y \le 17$

$-2x + 3y \le 6$

$y \ge 1$

$x, y \ge 0$

24. Solve the following LPP using Big-M method

Minimize $Z = 6x_1 + 4x_2 + 3x_3$

Subject to $4x_1 + 5x_2 + 3x_3 \geq 40;$

$2x_1 + x_2 + 6x_3 \geq 50;$

$3x_1 + 4x_2 + 2x_3 \geq 60;$

$x_1, x_2, x_3 \geq 0.$

25. Solve the following LPP using Two phase method

Minimize $Z = 4x_1 + x_2$

Subject to $3x_1 + x_2 = 3,$

$4x_1 + 3x_2 \geq 6,$

$x_1 + 2x_2 \leq 4,$

$x_1\, x_2 \geq 0.$

P.T.O.

OR

[Numerical Methods]

Group - A

Answer any ***ten*** of the following questions :

2×10=20

1. When Newton-Raphson method fails?

2. Prove that $\mu\delta = \frac{1}{2}\Delta + \frac{1}{2}E^{-1}\Delta$, symbols have their usual definition.

3. Define the degree of precision of the quadrature formula.

4. Write down the approximate representation of $\frac{2}{3}$ correct to four significant figures and then find : (i) Absolute error and (ii) relative error.

5. Define partial pivoting and complete pivoting.

6. State the round-off rule in numerical calculations.

7. Find the condition of convergence of fixed point iteration method.

8. Define percentage and truncation errors in numerical calculations.

9. Explain the concept of polynomial interpolation.

10. Define ill-conditioned problem.

11. What is prediction-correction method?

12. Construct a linear interpolating polynomial $f(x)$ with $f(1) = 3$ and $f(2) = -5$.

13. What is the sufficient condition for the convergence of Newton-Raphson method.

14. Round-off the following numbers correct up to five and four significant figures 0.4699987, 2.0046298, 0.000243468, 1.8948555.

15. The truncation error to compute the integration in Trapezoidal rule is of order

   (a) h

   (b) h^2

   (c) h^3

   (d) h^4

Group - B

Answer any ***four*** of the following questions :

5×4=20

16. Suppose you have to solve a cubic equation. Which method do you use and why? Explain the method, if any, and write down the order of convergence of this method.

17. Write down the quadratic polynomial which takes the

P.T.O.

same values as $f(x)$ at $x = -1, 0, 1$ and integrate it to obtain the integration rule

$$\int_{-1}^{1} f(x)\,dx \approx \{f(-1) + 4f(0) + f(1)\}/3$$

Assuming the error to have the form $Af^{iv}(\xi)$, $-1 < \xi < 1$, find the value of the constant A.

18. Explain Euler's method for solving a differential equation and give a geometrical interpretation of this method.

19. Solve the following system of equations by LU-factorization method :

$$x_1 + x_2 + x_3 = 4$$

$$2x_1 - x_2 + 3x_3 = 1$$

$$3x_1 + 2x_2 - x_3 = 1$$

20. Establish Newton's backward formula. When it is used?

21. Given $\sin 0° = 0$, $\sin 10° = 0.1736$, $\sin 20° = 0.3420$, $\sin 30° = 0.5000$ and $\sin 40° = 0.6428$.

(a) Find the value of $\sin 23°$

(b) Find the numerical value of $\frac{d^2y}{dx^2}$ at $x = 20°$ for $y = \sin x$.

Group - C

Answer any ***two*** of the following questions :

10×2=20

22. Explain the method of iteration for the numerical solution of an equation $f(x) = 0$ by re-setting it in the form $x = \phi(x)$. Find the condition of convergence of the method. Is it possible to formulate more than one iteration scheme for the equation? Formulate a convergent iteration scheme for solving the equation $2x - \sin x - 1 = 0$.

23. Explain the Gauss-Jacobi Method for solving a system of linear equations. Write the sufficient conditions for the convergence of the method. Present a Gauss-Seidel iterative scheme to solve the equations

    $5x + 2y + z = 0,\ x + 7y - 3z = 1,\ 2x + 2y - 7z = 2$

24. Establish the Newton-Cotes's formula for numerical integration without error term. Hence deduce the composite Trapezoidal rule from this formula. Also find the error term.

25. For equally spaced interpolating points $x_0, x_1, \ldots, x_n$, where $x_k = x_0 + kh$ $(h > 0,\ k = 0, 1, 2, \ldots, n)$ express $\Delta^k y_0$ in terms of the ordinates. Deduce the Newton's forward interpolation formula. Then derive the error expression in terms of $(n + 1)$th difference. Construct the interpolation polynomial for the function $y = \sin(\pi x)$, choosing the points $x_0 = 0$, $x_1 = 1/6$, $x_2 = 1/2$.

P.T.O.

OR

[Integer Programming and Theory of Games]

Group - A

Answer any ***ten*** questions : 2×10=20

1. Define analytical definition of saddle point.

2. Prove that the following payoff matrix has no saddle point

		B I	B II	B III
A	I	4	5	2
	II	1	4	6
	III	3	1	6

3. Show that the 2×2 game $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is non-strictly determined, if $a < b$, $a < c$, $d < b$ and $d < c$.

4. What are pure and mixed strategies?

5. Solve the following 2×2 game, the game being without saddle point using mixed strategies

		B B_1	B B_2
A	A_1	6	–4
	A_2	–1	2

6. A tosses two coins at a time. He receives Rs. 2 and Rs. 3 for two heads and two tails respectively and he losses Re. 1 for one head and one tail respectively. Find the value of the game.

7. Define a two-person zero sum or a rectangular game.

8. Define the optimal strategies and the value of the game. Are the optimal strategies unique?

9. Show graphically the feasible region of the following LPP

$$\text{Max } Z = x_1 + 2x_2$$

subject to $x_1 + x_2 \leq 2,$

$$x_1 + x_2 \geq 1,\ x_1, x_2 \geq 0.$$

10. Show that the set of vectors (1,0,0), (0,1,0), (0,0,1) and (1,1,1) are linearly dependent but any three of them are linearly independent.

11. Give two real examples of an mixed integer programming problem.

12. Write the drawback of branch and bound method.

13. What do you mean by Gomory's cutting plane method?

14. Give some applications of integer programming problem.

15. Write any two rules of dominance.

P.T.O.

Group - B

Answer any ***four*** questions : 5×4=20

16. Find the optimal solution to the following pay off matrix :

		B: B_1	B_2
A	A_1	2	7
	A_2	3	5
	A_3	11	2

17. Use dominance to reduce the following game problem to 2×2 game and hence find the optimal strategies and the value of the game

Player B

Player A	3	–2	4
	–1	4	2
	2	2	6

18. Transform the games with pay-off matrices to the corresponding L.P.P.

B

A	2	–2	3
	–3	5	–1

19. Find the optimal solution to the I.P.P.

Maximize $Z = x_1 + x_2$

subject to $3x_1 + 2x_2 \leq 5;$

$x_2 \leq 2;$

$x_1, x_2 \geq 0$ and all are integers.

20. Prove that if we add a fixed number X to each element of the payoff matrix, then the optimal strategies remain unchanged while the value of the game is increases by X.

21. Derive the Gomory's constraint of the optimal table of a general LPP.

Group - C

Answer any *two* questions : 10×2=20

22. Use Gomory's cutting plane method to find the optimal solution of the I.P.P.

Maximize $Z = 2x_1 + 2x_2$

subject to $5x_1 + 3x_2 \leq 8$

$x_1 + 2x_2 \leq 4$

$x_1, x_2 \geq 0$ and all are integers.

P.T.O.

23. Use branch and bound technique to solve the following integer programming problem

Minimize $Z = 4x_1 + 3x_2$

subject to $5x_1 + 3x_2 \geq 30;$

$x_1 \leq 4;$

$x_2 \leq 6;$

$x_1, x_2 \geq 0$ and all are integers.

24. For the following payoff table, transform the zero-sum game into an equivalent linear programming problem and solve it by simplex method

		Player B		
		B_1	B_2	B_3
	A_1	–1	2	1
Player A	A_2	1	–2	2
	A_3	3	4	–3

25. Find the optimum integer solution of the following I.P.P.

Minimize $Z = 9x_1 + 10x_2$

subject to $4x_1 + 3x_2 \geq 40;$

$x_1 \leq 9;$

$x_2 \geq 8;$

$x_1, x_2 \geq 0$ and all are integers.
